Analysis of the near-source error in TEM due to the dipole hypothesis

Zhou Nan-Nan a,⁎, Xue Guo-qiāng a, L.-J. Gelius b, Wang He-yuan c, Yan Shu d

a Key Laboratory of Mineral Resources, Institute of Geology and Geophysics, Chinese Academy of Sciences, Beijing 100029, China
b Department of Geosciences, University of Oslo, 0316 Oslo, Norway
c College of Sciences, Liaoning University of Technology, Jinzhou 121001, China
d School of Computer Science and Telecommunication Engineering, Jiangsu University, Zhenjiang 212013, China

A R T I C L E   I N F O

Article history:
Received 26 August 2014
Received in revised form 11 February 2015
Accepted 20 February 2015
Available online 27 February 2015

Keywords:
Dipole hypothesis
Point charge
Damped wave equation
Transient electromagnetic response
Mine TEM

A B S T R A C T

The dipole hypothesis commonly used in the description of electromagnetic fields associated with TEM loop antennas or grounded wire source is not accurate enough in the near-source zones where the response is also of significant importance for clearly mapping subtle structures embedded in shallow settings with geologic complexities. In order to better understand these near-source errors, we investigate the scale of errors resulting from the direct-dipole and superposing-dipole approximation of the large loop or long grounded wire sources. The results demonstrate that both the direct-dipole approximation and the superposing-dipole approximation fail to give accurate results in the near-source zone. Not only the direct-dipole approximation but also superposing-dipole approximation will arouse some errors. Therefore, a 'point-charge' solution of the EM damped wave equation is proposed and derived to calculate the near source responses. Forward modeling and field data analysis demonstrate that the proposed method provides more accurate results in the near source areas and can be effectively used to calculate the TEM response in the near source region.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

The square loop and the grounded wire are commonly used as the transmitting antenna in TEM sounding. Simulation of TEM data is often based on the dipole hypothesis of these large-scale sources.

For the grounded-wire source, the electromagnetic field is excited by a grounded-wire source which is often considered as the electric dipole (Kaufman and Keller, 1983). The LOTEM method is one of the most popular configurations of grounded source TEM method because the dipole hypothesis is reasonable and can be applied to the modeling and interpretation of LOTEM data. Nevertheless, the signal strength and the resolution of LOTEM method are limited due to the decay properties of the diffusive electromagnetic field with the receiver-transmitter offset. In recent years, the near-source or small-offset TEM detecting technology has been studied and becomes the focus of recent research. Xue et al. (2013) pointed out that the small-offset time-domain or transient electromagnetic method has the merit of a strong signal level which enhances deep-detecting capability and high accuracy. However, unlike in the case of LOTEM method, the dipole hypothesis is invalid in near source configuration. The TEM surveys aimed at resolving relatively small structures embedded within settings of increasingly geologic complexities, which may include short distances at which the error of the dipole approximation is on the same level as (or larger than) the target anomalies. Therefore, such approximation using direct dipole hypothesis in near-source or small-offset region will arouse huge errors.

In order to decrease this error, dipole superposition method is put forward to calculate the response of grounded-wire source TEM method. Stoyer (1990) adopted 27 segments to calculate vertical magnetic field and analyzed the error between response of grounded wire and that of electric dipole. Streich and Becken (2011) analyzed the error between dipole source and finite-length wire in frequency domain.

For loop source, near-source and far-source data set are measured in large fixed-loop transient electromagnetic method. The dipole hypothesis is reasonable in far-source position (outside the loop) while the hypothesis cannot be applied in near-source (in loop) response because the dipole hypothesis may be not fully satisfied in the region near the transmitting wire of the loop (Streich and Becken, 2011). Hence, dipole superposition method is used to calculate more accurate solution of large loop TEM. Ward and Hohmann (1987) proposed that one should divide the rectangular loop into an ensemble of small rectangular surface elements. Each of these elements is approximated by a magnetic dipole and the total field response is obtained by integration along the loop surface. Poddar (1983) divided each loop side into small current sections, which may then be thought as point electric dipoles and calculated the overall response in the frequency domain by integrating along the loop. Furthermore, Raiche (1987) applied the nested interpolation method to obtain the field solution of the polygonal loop. Goldman and Fitterman (1987) used the integral of the electric dipole to deduce the surface field excited by the rectangular loop. Li et al. (2010) also represented the loop as the magnetic dipole, and obtained more realistic expressions of the electric and magnetic fields. Based on these results, the symmetrical relationship between electromagnetic and magnetic.
fields may be further analyzed. Li et al. (2007) approximated the electric dipole integral by a discrete summation, and obtained an expression for the apparent resistivity valid for an arbitrarily shaped TEM loop source. Xue et al. (2012) carried out field exploration in coal mine employing a modeling technique similar to that of Poddar (1983).

The superposition methods described above help to analyze the TEM method and can be used as an approximation in the near source TEM method. However, even the superposing-dipole approximation cannot give an accurate description of the near-source responses. The calculation error brought by the length of the source is inevitable. Therefore, we propose a new infinitesimal to replace the Hertzian dipole with ‘point-charge’. When compared with the Hertzian dipole, this new incremental ‘point-charge’ approach seems to give more physically accurate results close to the transmitting antenna.

2. Illustration of the errors due to direct-dipole approximation of large-scale source

We first analyze the approximation errors due to the direct-dipole hypothesis through the comparison between long grounded-wire source and electric dipole. Compared to other electromagnetic components, only the time derivative of vertical magnetic field has the analytical expression in case of homogeneous half space. Therefore, the time derivative of vertical magnetic field is used to analyze the approximation errors.

Expression of the time-derivative of vertical magnetic field induced by grounded-wire source is given by Ward and Hohmann (1987):

\[ \frac{\partial h_y}{\partial t} = \frac{2I}{\mu_0 \pi_0 \sigma} \left( (1 + i2\theta) e^{-i2\theta} \operatorname{erf}(i2\theta) - \frac{1}{R} \left( 1 - \frac{y^2}{2R^2} \right) \operatorname{erf}(i\theta) + \frac{i\theta y^2}{\sqrt{\pi} R^2} e^{-i\theta^2} \right) \]

where \( \theta = (\mu_0 i2R)^{1/2} = (y^2 + L^2)^{1/2} \). L is half of the line. The x-axis is oriented along the wire, with the origin at the center of the wire. The y-axis is perpendicular to the wire on the surface of the earth, and the z-axis is downward. R is the receiver-source distance and \( \sigma \) is the conductivity of the earth.

Similarly, expression of the time-derivative for vertical magnetic field induced by electric dipole source is given by Ward and Hohmann (1987):

\[ \frac{\partial h_y}{\partial t} = \frac{I ds}{2 \mu_0 \sigma R^3} \left( 3 \operatorname{erf}(\theta R) - \frac{2}{\sqrt{\pi}} 3(2 + 2\theta R^2) e^{-\theta^2 R^2} \right) \]

where ds represents the length of the dipole source and is equal to 2L.

Calculation of the time derivative for vertical magnetic field induced by grounded-wire and electric dipole is based on Eqs. (1) and (2) respectively. The modified calculation formula of relative error between both sources is evaluated as:

\[ \text{ERR} = 2 \times \frac{\text{ABS}(V_p - V_k)}{(V_p + V_k)} \]

where \( V_p \) is the value of grounded wire, and \( V_k \) is the value of an electric dipole.

The error distribution at different times is shown in Fig. 1.

As shown in Fig. 1, the error along the y-axis is smaller than that of other directions. The error along the x-axis is always very large. For instance, if an error of 5% is considered acceptable, only the region within the 10° angle of the y-axis will be reasonable for us to use the dipole to approximate the grounded wire. At the time of 1e-6s, when the offset is far larger than the source length, the error along the y-axis could be acceptable. The offset along y-axis that can be acceptable will be larger with increasing time. At the time of 1e-2s, this offset is 800 m far larger than the source size. The direct dipole hypothesis of long grounded-wire source will lead to huge errors in most of the displayed region.

Fig. 2 shows the decay curves of the induced voltage from grounded wire and electric dipole at different points. As shown in Fig. 2, the separating time of the two curves becomes larger and the error becomes smaller with the increasing offset. At point (0, 21), there is no coincident position in the two curves and the error becomes larger with time. At point (0, 111), the two curves coincide at early time and difference occurs at time of 1e-5s. When the offset is smaller than or equal to the source size, the direct-dipole hypothesis of the grounded wire is acceptable only at the very early time. With the increase of the offset, the time range in which the direct-dipole hypothesis is reasonable becomes larger.

As shown in Fig. 3, the error curves between these two sources under different resistivity have apparent distinction at early time. At early time, the value of relative difference with the resistivity of 100 \( \Omega \cdot m \) is far larger than with the resistivity of 1 \( \Omega \cdot m \). The error becomes the same at late time. In conclusion, the effect of the resistivity to the error caused by the direct-dipole hypothesis can be only ignored at late time.

To sum up, we analyzed the relative error of the response between grounded wire and electric dipole from aspects of distribution, resistivity and offset. Only when the receiver points are in the range that at the angle 10° from the y-axis and the offset is large enough, direct-dipole hypothesis is applicable. For most of the region, this hypothesis will undesirably generate huge errors.

3. Illustration of the errors due to superposing-dipole approximation of large-size source

In order to decrease or eliminate the error caused by the direct-dipole approximation, many scholars have put forward the method of dipole superposition. Taking large-loop TEM for example, the loop can be divided into small surface pieces or line segments, and every part can be considered as the magnetic or electric dipole. The integration is to subdivide the integral interval and integrate using different methods, which is similar to the dipole superposition. So, we can use the dipole superposition to replace the surface or line integral. We use the proportionality coefficient between the offset and the length of line segment to achieve the superposition of dipole.

As shown in Fig. 4, every side of the loop is divided into small electric dipoles. The length of the dipole is ds, the proportionality coefficient is m and the number of the dipole divided by every side is n. n = li / ds, where li is the length of the i side of the loop, r is the receiver-source distance, d is the vertical distance between receiver point P and ds. The response of the rectangular loop can be calculated using following equation:

\[ f_{\text{loop}}(x, y) = \sum_{i=1}^{4} f_{i}(x, y) \]

where, \( f_{\text{loop}} \) is the response induced by the loop, \( f_{i}(x, y) \) is the response induced by every side and \( f_{i}(x, y) \) is the response induced by each dipole.

It is well known that the response at the central point of rectangular loop is quite the same with that at the central point of circular loop with the same area (Nabighian, 1992). The response has the analytical solution at the central point of circular loop on the surface of homogeneous earth. Hence, the analytical solution of the vertical magnetic field at the central point of the loop can be used to calculate the response at
the central point of rectangular loop. The expression of the vertical magnetic field at the central point of the circular loop is

\[ h_z = \frac{I}{2\pi} \left[ \frac{3}{\pi^{1/2}a^2} e^{-\theta^2} + \left(1 - \frac{3}{2\pi^2a^2}\right) \text{erf}(\theta) \right] \]

where \( \theta = \left(\frac{2\pi \sqrt{a}}{a}\right) \), \( \text{erf} \) is the error function.

Use Eqs. (4) and (6) to calculate the response at the central point of rectangular loop and circular loop. Taking the response from the circular loop as the benchmark response, the apparent resistivity can be calculated using the iteration method. The resistivity of the homogenous earth is 100 Ω·m. Fig. 5 shows the calculated apparent resistivity with different proportionality coefficients. As shown in Fig. 5, when the coefficient is small, apparent resistivity is far larger than the real resistivity. The apparent resistivity gets close to the 100 Ω·m with the increasing coefficient. When the coefficient is larger than 50, the apparent is the same with the real resistivity. So, it is concluded that the dipole superposition is effective for the calculation of the response of rectangular loop when the coefficient is large enough. However, the difference will not disappear with the increasing coefficient. For example, even when the coefficient is larger than 50, there is still some errors caused by the scale of the dipole. Using the response of the rectangular loop with the coefficient, 82 as the benchmark response, calculate the relative difference between the responses with different coefficients. Fig. 6 displays two comparison results among the responses calculated using different coefficients, 50, 82 and 100. As shown in Fig. 6, the relative difference is still larger than 1%, which cannot be ignored for high-resolution exploration.

4. Direct time-domain electromagnetic field of point charge in homogenous conductive full space

4.1. Physical mechanism of point charge

An electric dipole consists of two hetero-charges with equal magnitude. When the dipole antenna is charged with alternating current, the accelerating movement of the charges creates an electromagnetic field. Theoretically, radiation is the spread of electromagnetic field excited either by the accelerating movement of point charge or by a fixed-position point charge with variable magnitude or polarity. Therefore, the radiation of the electric dipole is the special case of the radiation of an accelerating charge (Griffiths, 1999) and the infinitesimal point charge is the more basic source for producing electromagnetic field (Rodney, 1994). Based on the description above, we seek to find the direct time-domain solution of the electromagnetic field generated by
an infinitesimal point charge instead of the dipole source. The solution of transient field can be directly derived from the time-domain auxiliary potential function without the transformation from frequency to time domain.

4.2. Direct time-domain Green function

In a homogeneous and isotropic medium, Maxwell functions are (Kaufman and Keller, 1983)

\[ \nabla \times \mathbf{H} = J + \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \]  

\[ \nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \]  

\[ \nabla \cdot \mathbf{H} = 0 \]  

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon} \]  

where \( \mathbf{J} \) denotes the vector of current source density, \( \rho \) presents the density of source point charge, \( \mathbf{E} \) presents electric field intensity (V/m), \( \mathbf{H} \) denotes magnetic field intensity (Wb/m²), \( \nabla^2 = \frac{1}{c^2} \) is the light speed, \( \mu \) and \( \varepsilon \) are the magnetic permeability and electric permittivity respectively.
Based on Eqs. (7a)-(7d), damped wave vector equations of electromagnetic field are (Kaufman and Keller, 1983):

\[
\nabla^2 E - \alpha \frac{\partial E}{\partial t} - \mu_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial J}{\partial t} + \nabla \left( \frac{P}{\varepsilon} \right) \tag{8a}
\]

\[

\nabla^2 H - \alpha \frac{\partial H}{\partial t} - \mu_0 \frac{\partial^2 H}{\partial t^2} = -\nabla \times \mathbf{J}. \tag{8b}
\]

Green function is the response of the centralized source in 3D conductive medium. The damped wave equation in three dimensions is:

\[
\left( \nabla^2 - \mu_0 \frac{\partial^2}{\partial t^2} - \mu_0 \alpha \frac{\partial}{\partial t} \right) g(x,y,z,t) = -\delta(x)\delta(y)\delta(z)\delta(t). \tag{9}
\]

The solution of the Eq. (9) is given by Weng (1995):

\[
g(x,y,z,t) = \frac{e^{-\frac{x^2}{2\sigma^2}}}{4\pi \sigma^2} \cdot u(\sqrt{t^2 - r^2})I_0 \left( \frac{1}{2\sqrt{t^2 - r^2}} \right). \tag{10}
\]

Next, using the differential calculation

\[
\frac{\partial}{\partial t} \left[ u(\sqrt{t^2 - r^2})I_0 \left( \frac{1}{2\sqrt{t^2 - r^2}} \right) \right] = \delta(\sqrt{t^2 - r^2})I_0 \left( \frac{1}{2\sqrt{t^2 - r^2}} \right) + t_1 \left( \frac{1}{2\sqrt{t^2 - r^2}} \right) \frac{1}{2\sqrt{t^2 - r^2}}. \tag{11}
\]

As a result, Eq. (10) becomes

\[
g(x,y,z,t) = \frac{e^{-\frac{x^2}{2\sigma^2}}}{4\pi \sigma^2} \delta(\sqrt{t^2 - r^2}) \cdot I_0 \left( \frac{1}{2\sqrt{t^2 - r^2}} \right) \frac{1}{2\sqrt{t^2 - r^2}} \cdot u(\sqrt{t^2 - r^2}). \tag{12}
\]

Letting \( \alpha = \alpha, \sigma = \sigma, \) and with the identity

\[
e^{\frac{x^2}{2\sigma^2}} \left( \frac{1}{2\sqrt{t^2 - r^2}} \right) = \frac{1}{\sqrt{t^2 - r^2}} = e^{-\alpha t/c} \]

we arrive at

\[
g = \frac{1}{4\pi} \left( \delta(\sqrt{t^2 - r^2})e^{-\alpha t/c} + t_1 \left( a\sqrt{t^2 - r^2}/c^2 \right) \right) \frac{1}{\sqrt{t^2 - r^2}}. \tag{13}
\]

The left part will not be zero only at the time of signal arrival and the right part will be zero in insulating medium. Eq. (13) is the time-domain Green function in conductive space.

With

\[
F = \int_{0}^{t} G(x, y, z, t) \right| Q(x, y, z, t) \right| dx dy dz dt
\]

**Fig. 4.** Sketch map of the loop and divided dipole.

**Fig. 5.** Calculated apparent resistivity with different proportionality coefficients.

**Fig. 6.** Relative difference among the responses with different coefficients, 50, 82 and 100.

**Fig. 7.** Electric field comparison vs time at \( r = 100 \) m.
the potential function for damped wave equation is

$$\mathbf{F} = \int_0^t \int_{4\pi r^2} \frac{\delta(t-r/c) e^{-\sigma r/c}}{4\pi r^2} + I_1 \left( \alpha \sqrt{t^2-r^2/c^2} \right) \mathbf{E}(x, y, z, t) dxdydz dt.$$  \hspace{1cm} (13)

$$\times \frac{c}{z} e^{-\sigma r} \frac{1}{\sqrt{t^2-r^2/c^2}} \cdot u(t-r/c) \mathbf{Q}(x, y, z, t) dxdydz dt.$$

4.3. Direct time-domain electric field of point charge

Although the direction of the magnetic moment of the point charge may be arbitrary, the point source is one part of the real source and has the assigned-direction magnetic moment. This assumption will weaken the calculation difficulty.

Letting the potential function for point charge in z-direction, $\mathbf{F}_z$ and using

$$\mathbf{E} = \nabla \times \mathbf{F}_z$$

we arrive at

$$E_{\varphi} = \nabla \times \mathbf{F}_z = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r \mathbf{F}_r) - \frac{\partial \mathbf{F}_\theta}{\partial \theta} \right].$$  \hspace{1cm} (14)

In spherical coordinate,

$$\mathbf{F}_r = F_r \cos \theta$$
$$\mathbf{F}_\theta = -F_\theta \sin \theta.$$

Substitute Eq. (15) into Eq. (14), then

$$E_{\varphi} = -\frac{\partial \mathbf{F}_r}{\partial r} \sin \theta. $$  \hspace{1cm} (16)

According to the Eq. (13), impulsive function $\delta(t - r/c)$ is not zero at $t = r/c$ and step function $u(t - r/c)$ has the value when $t$ is larger than $r/c$. So, the solution of $E_{\varphi}$ can be separated into two parts:

1) $$E_{\varphi 1} = \int_0^t \int_{4\pi r^2} \frac{\delta(t-r/c) e^{-\sigma r/c}}{4\pi r^2} Q(x, y, z, t) dxdydz dt, \hspace{1cm} t = r/c.$$  \hspace{1cm} (17)

After derivation,

$$E_{\varphi 1} = \int_0^t \int_{4\pi r^2} \left[ 1 + \frac{\alpha r}{c} \delta(t-r/c) + \frac{\alpha r}{c} \delta(t-r/c) \right] e^{-\sigma r/c} Q(x, y, z) dxdydz. \hspace{1cm} t = r/c.$$  \hspace{1cm} (18)

2) $$E_{\varphi 2} = \int_0^t \int_{4\pi r^2} \frac{1}{4\pi r^2} I_1 \left( \alpha \sqrt{t^2-r^2/c^2} \right) \mathbf{E}(x, y, z, t) dxdydz dt. \hspace{1cm} t = r/c.$$  \hspace{1cm} (19)

By using the differential form of modified Bessel function

$$I_1 = \frac{1}{z} (I_0 + I_2) \text{ and } I_{n+1} = -\frac{2n}{x} I_n(x), \hspace{1cm} n = 1, 2, 3, ...$$

we arrive at

$$E_{\varphi 2} = \int_0^t \int_{4\pi r^2} \frac{1}{4\pi r^2} I_1 \left( \alpha \sqrt{t^2-r^2/c^2} \right) e^{-\sigma r/c} Q(x, y, z) dxdydz + \frac{I_1}{c} Q(x, y, z) dxdydz dt. \hspace{1cm} t = r/c.$$  \hspace{1cm} (20)

where, $\frac{1}{\gamma^2} = \alpha, \hspace{0.2cm} c = \frac{1}{\sqrt{\mu \varepsilon}}$ is the velocity of light; $\sigma$ is the conductivity; $\mu$ is the magnetic permeability; $\varepsilon$ is the electric permittivity. $I_n + 1$ is the $n + 1$ order modified Bessel function.

5. Comparison of the response between point-charge infinitesimal and dipole in homogenous full space

Kaufman and Keller (1983) presented the expression of $E_{\varphi}$ under the quasi-static condition using integral of the response from dipole:

$$E_{\varphi} = \sqrt{2 \frac{M \varepsilon}{\pi 4 \pi r^2}} \sin \theta e^{-t^2/2} \sin \theta.$$  \hspace{1cm} (21)

where, $\mu = \frac{\sqrt{\mu \varepsilon}}{c}$.

Letting $\sigma = 0.015/m, \varepsilon = \varepsilon_0 = 8.85 \times 10^{-12} F/m, M = 1$, use Eqs. (20) and (21) to calculate the electric field.

Fig. 7 shows the electric field comparison curves at 100 m. Solid line represents the traditional quasi-static solution and dashed line represents the direct time-domain solution. There is an apparent difference at early-time stage.

Fig. 8 shows the electric field comparison curve at time 1e6s. As shown in the figure, the curve for traditional solution changes more rapidly than that for direct time-domain solution. At early-time stage, there is an apparent difference between these two solutions. With the fast development of VETEM (USA) and ATTEM (China), the research of exact solution at the very early time is of great importance.

Let the radius of the magnetic dipole 1 m and current 1 A. To simplify the calculations, we assumed the unit magnetic moment for the infinitesimal point charge and the dipole sources also equal to one. Then, we compared the fields of these two sources in the all zones. The relative errors between these two sources can be calculated using Eq. (3) and the calculated errors are shown in Fig. 9.
As shown in Fig. 9, the larger the distance is, the smaller the relative errors are. The relative errors between the infinitesimal point charge and magnetic dipole are less than 1% when the distance is larger than 10 m. The dipole hypothesis will lead to huge errors in near-source zone.

6. Response induced by rectangular loop based on point-charge infinitesimal in homogeneous full space

In mine TEM method, the response of the loop is calculated under the assumption of homogeneous full space. So, the derivation of response in homogeneous full space will be of great importance. For the rectangular loop, the response is usually calculated by the superposition or integral of the response from the magnetic or electric dipole (Nabighian, 1992), which is suitable for the calculation of magnetic field, but not suitable for the calculation of induced voltage. Here, introduce the law of reciprocity to calculate the induced voltage from rectangular loop. By the law of reciprocity, the same induced voltage will be generated in the rectangular loop if the currents were flowing in the magnetic dipole, their respective positions remaining the same. So, if the induced voltage received in the rectangular loop from the magnetic dipole is calculated, the induced voltage received in the magnetic dipole from the rectangular loop is also known. Using the azimuthally electric field parallel to any side of the rectangular loop, viz., \( E_\phi \sin \theta \) to integrate along four sides; the induced voltage \( V \) received in the loop from the dipole is given (shown in Fig. 10):

\[
V = A + B + C + D
\]

(22)

where, \( A, B, C, \) and \( D \) represent the response of the four sides. and

\[
A = -(b-y) \int_{-a}^{a} E_\phi \sin \theta dx' \\
(22 - 1)
\]

with

\[
r = \sqrt{(x' - x)^2 + (b - y)^2}
\]
and
\[ B = -(a-x) \int_{-b}^{b} E_c \sin \theta dy' \]  \hspace{1cm} (22-2)

with
\[ r = \sqrt{(a-x)^2 + (y' - y)^2} \]

and
\[ C = -(b+y) \int_{-a}^{a} E_c \sin \theta dx' \]  \hspace{1cm} (22-3)

with
\[ r = \sqrt{(x' - x)^2 + (b + y)^2} \]

and
\[ D = -(a+x) \int_{-b}^{b} E_c \sin \theta dy' \]  \hspace{1cm} (22-4)

with
\[ r = \sqrt{(a+x)^2 + (y' - y)^2} \]

\((x, y)\) is the coordinate of observation point; \((x', y')\) is the coordinate of transmitting point in the rectangular.

Substituting Eqs. (21) and (20) into Eq. (22), the induced voltage excited by the rectangular loop based on new formula of point charge and traditional formula of dipole is shown in Fig. 11.

As shown in Fig. 11, there is great difference between these two calculated voltages at early time. These two curves are merged together with time. However, there is still little differences between these two curves, which cannot be displayed in log–log plot. The response calculated using new equation and new source is more accurate than the traditional source.

7. Field survey of mine TEM

One mine TEM survey is conducted in one coal mine in Shanxi Province. Fig. 12 shows the sketch map of the survey. Australia-made Terra-TEM is used to detect the coal seam in this survey. For the limited space in the tunnel, the multi-turn loop is adopted as the transmitting loop and SB-70K magnetic probe as the receiving device. The transmitting area and receiving area are 160 m² and 100 m² respectively. Transmitting current is 5 A and turn-off time is 1.034 ms for the effect of damping resistors of multi-turn loop. 42 time gates are used (Table 1). The observed decay curve of induced voltage at central point of the transmitting loop is shown in Fig. 13. Use the method of Eaton to correct the turn-off effect. For the full-space mine TEM, the observed induced voltage will be smaller while the induced voltage will be larger in half-space TEM after the turn-off correction (Eaton and Hohmann, 1987). The corrected decay curve is also shown in Fig. 13. According to the drilling result, the resistivity of the coal seam is 70 Ω·m. Use Eq. (22) to calculate the induced voltage based on the electric field of point-charge and magnetic dipole with Eqs. (20) and (21). The forward results are shown in Fig. 14.

As shown in Fig. 14, there is obvious difference between observed voltage and forward results at early time. The difference becomes smaller with time. Compared to the forward curve using traditional formula, the forward result using new formula is more close to the observed decay curve. It is concluded that the new point-charge source is more basic and the response using new formula is more accurate.

8. Conclusions

The near-field transient electromagnetic method has the merits of a strong signal level, deep penetration depth and high accuracy. Traditionally, dipole theory has been employed in the study of time-
domain electromagnetic field responses associated with a large-loop antenna and grounded-wire source. However, it is well known that far-field theory as represented by such a dipole is not applicable when used for describing the near-field measurements. Thus in the case of a large-loop and short-offset grounded-wire TEM source, current modeling techniques do not match well with the actual field measurements especially in the near source region. An extensive error analysis has been conducted in this paper to support this conclusion. The direct-dipole approximation will arouse huge errors in near-source zone. Superposing-dipole method can help decrease this error but cannot eliminate error.

The ‘point-charge’ solution of the damped wave equation is proposed to describe the antenna response on an incremental scale. When compared with the point-dipole approach, this new technique seems to give more accurate result in near-source zones, which is verified by both forward modeling and field data. This analysis implies that the ‘point-charge’ elementary solution can be effectively used to calculate TEM response in the near-source regions.

Acknowledgments

This research was supported by the R&D of Key Instruments and Technologies for Deep Resources Prospecting (the National R&D Projects for Key Scientific Instruments), Grant No. ZDYZZ2012-1-04-05, the State Major Basic Research Program of the People’s Republic of China (2012CB416605) and the Natural Science Foundation of China (NSFC) (41174090, 41174108, and 41130419).

References