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THEORETICAL ANALYSIS AND NUMERICAL CALCULATION OF LOOP-SOURCE TRANSIENT ELECTROMAGNETIC IMAGING

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Abstract In order to improve the prospecting precision to the earth by the transient electromagnetic method (TEM), We propose an idea of TEM imaging technology and numerical calculation method. The transient electromagnetic response of the layered earth is discussed in the frequency domain, then a double integral in which the kernel is unknown is obtained. The solution of the electromagnetic field in the medium of the layered earth is separated into the form of upper and down traveling wave. Then a linear equation set in which reflection coefficients are unknown is derived. The numerical solution of wave-impedance and reflection coefficients is given as follows: Transforming the magnetic field data from time into the frequency domain, replacing the kernel of equation with the equivalent wave-impedance of homogeneous half-space, getting the equivalent wave-impedance in the frequency domain after digital filter, then returning the equivalent wave-impedance into the time domain, constructing equations with it getting reflection coefficients from the equation with the method of linear programming in the time domain. The imaging result section can be drawn. The imaging result from the theory model shows that the method put forward in this paper can improve greatly the ability to recognize underground electric interfaces.

Key words Loop source, Numerical calculation, Wave-impedance, Reflection coefficients, Digital filter, Linear programming, Transient electromagnetic image.

1 INTRODUCTION

The transient electromagnetic method (TEM) is widely used in geophysical exploration because of its advantage of portable configuration and high resolution. With the increasing difficulties of resource exploration and requirement to improve engineering investigation precision. It is important to further enhance TEM interpretation level and distinguish ability[1]. The large-loop transient electromagnetic method uses a large loop or square loop to survey in the center of the square loop. This kind of configuration has strong transmitting energy and stable field signal. It is suitable for engineering survey and mineral source exploration of high precision requirement.

An underground target can be detected according to the properties of electromagnetic wave during the propagation through media. The idea of the magneto telluric pseudo-seismic interpretation method has been put forward according to the unanimity of reflection coefficient function and similarity of propagation in elastic media between electromagnetic wave and seismic wave[2]. According to the electromagnetic theory study, the magnetic field direction is perpendicular to the direction of plane of loop horizon at the center of the loop where the distribution of field is uniform. The electromagnetic field can be regarded as a plane wave at the center of the loop. There is similarity between electromagnetic wave and plane electromagnetic wave. The propagation features of plane electromagnetic wave and seismic reflection wave are analyzed in detail under the condition of homogeneous media, then transient electromagnetic field data are processed by time-frequency transformation on the basis of previous studies. The time domain electromagnetic field decay signal is transformed into frequency domain electromagnetic signal. The relationship of frequency and time can be expressed as: $f = (194 \sim 200)/t$, which is used to transform TEM field data to plane wave field data, then the magnetotelluric pseudo-seismic interpretation method is used to process TEM data, and fast pseudo-seismic interpretation of electromagnetic sounding data is realized[1]. According to the wave-field transformation method, through U transformation which changes electromagnetic field question of losing medium to the question of wave field of the medium of dispersion, at last long-offset transient electromagnetic migration imaging is arrived[3].

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At present, there are many reports about loop magnetic source near-area transient electromagnetic sounding imaging. The aim of this paper is to describe how to directly use large loop transient electromagnetic field data to get time domain electromagnetic field transmission function, and then get the solution of underground reflection coefficients. We build a system of the imaging method and numerical analytical method. The equivalent wave-impendence field magnetic data are obtained using linear digital filter technology, and with the known reflection coefficients, reflection coefficients of every mini-layered are gotten by the method of linear programming. At last TEM imaging is finished.

2 BUILDING THE MATHEMATICAL MODEL OF TRANSIENT ELECTROMAGNETIC IMAGING

There are two key equations of transient electromagnetic imaging. One is the TEM response equation of layered medium; another is the equation set that contains reflection coefficients

2.1 TEM Response Equation of Horizontal Layered Medium

Maxwell equations give the basic rule of electromagnetic field[4]

$$\nabla \times \boldsymbol{E} = -\frac{\partial B}{\partial t}, \tag{1}$$

$$\nabla \times \boldsymbol{H} = j_{\mathrm{e}} + \frac{\partial D}{\partial t}, \tag{2}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{3}$$

$$\nabla \cdot \boldsymbol{D} = \rho, \tag{4}$$

where ∇ stands for the Laplace calculator, $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic induction intensity, $\boldsymbol{H}$ is the magnetic field intensity, $\boldsymbol{D}$ is the electric displacement vector, j is the current density, ρ is the free charge density.

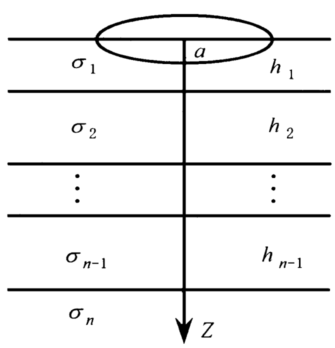

Fig. 1 Schematic drawing of a loop source on the surface of the layered earth

The model of earth current is shown in Fig. 1, the loop radius is a, current is I, circle-loop is laid on the surface of a n-layer medium levelly, the resistivity is $\sigma_1, \sigma_2, \cdots, \sigma_n$, we build a cylindrical coordinate system, the zero point of coordination is at the center of loop, the thickness of each layer is $h_1, h_2, \cdots, h_n$. In order to make calculation easy, we introduce a scalar potential function F to the study.

Let

$$H_z = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right), \tag{5}$$

$$E_\phi = \mathrm{i}\omega\mu\frac{\partial F}{\partial r}, \tag{6}$$

where H_z is the vertical component of magnetic field, E_ϕ is the tangential component of electric field, r is field radial vector, ω presents wave frequency, and μ presents magnetic permeability of medium.

From Maxwell equations, we know that the scalar potential function F satisfies the following Hemholtz equation:

$$\nabla^2 F - k^2 F = 0, \tag{7}$$

where k is the wave number. In the j-th layer medium of the earth, the general answer of F is

$$F_j = \frac{I_0 a}{2}\int_0^\infty \frac{\mathrm{J}_1(\lambda a)}{\lambda}[a_j \mathrm{e}^{-u_j a} + b_j \mathrm{e}^{u_j z}]\mathrm{J}_0(\lambda r)\mathrm{d}\lambda, \tag{8}$$

where $\mathrm{J}_0(\ )$ and $\mathrm{J}_1(\ )$ are Bessel functions, $u_j=\sqrt{\lambda^2+k_j^2}$, λ is the integral parameter, σ_j is conductivity, wave number $k_j=(-\mathrm{i}\omega\sigma_j\mu_0)^{1/2}$, n is layer number, μ_0 is magnetic permeability of vacuum.

In order to determine unknown coefficients a_j and b_j in Eq.(8), we must use the boundary conditions. In the case of the one dimensional layered earth, the tangential component of electric field and the vertical component of magnetic induction intensity are continuous, So function F can be calculated by the boundary condition, then every electromagnetic field component can be obtained from function F accordingly. In the center of the loop electric and magnetic field components can be written as[4]

$$H_z=I_0a\int_0^\infty\frac{\lambda Z^{(1)}}{Z^{(1)}+Z_0}\mathrm{J}_1(\lambda a)\mathrm{d}\lambda. \tag{9}$$

Magnetic field vertical component.

Component of magnetic induction intensity

$$\frac{\mathrm{d}B_z(\omega)}{\mathrm{d}t}=-\mathrm{i}\omega\mu_0I_0\int_0^\infty\frac{\lambda Z^{(1)}}{Z^{(1)}+Z_0}\mathrm{J}_1(\lambda a)\mathrm{d}\lambda, \tag{10}$$

where I is the electric current, $\mathrm{J}_1(\lambda a)$ is the First-order Bessel function, $\frac{\mathrm{d}B(\omega)}{\mathrm{d}t}$ is differential parameter of magnetic field, $Z^{(1)}$ is surface wave impedance of horizontally layered medium of some frequency, and Z_0 is the characteristic wave impedance of free space. When excited by step-current, the original magnetic field of the loop is

$$H_0(t)=\begin{cases}H_0, & (t<0),\\ 0, & (t>0).\end{cases} \tag{11}$$

From the theory of frequency-spectrum analysis, the transient electromagnetic response in time domain can be written as[4]

$$H_z(t)=\frac{2}{\pi}\int_0^\infty\mathrm{Im}\left[I_0a\int_0^\infty\frac{\lambda Z^{(1)}}{Z^{(1)}+Z_0}\mathrm{J}_1(\lambda a)\mathrm{d}\lambda\right]\frac{\cos\omega t}{\omega}\mathrm{d}\omega, \tag{12}$$

$$\frac{\mathrm{d}B(t)}{\mathrm{d}t}=\frac{2}{\pi}\int_0^\infty\mathrm{Re}\left[I_0a\int_0^\infty\frac{\lambda Z^{(1)}}{Z^{(1)}+Z_0}\mathrm{J}_1(\lambda a)\mathrm{d}\lambda\right]\cos\omega t\mathrm{d}\omega. \tag{13}$$

where Im is the part of imaginary and Re is the real part. So, Eq.(13) shows clearly that transient electromagnetic response of the layered earth can be written as a double integral. It is a two binary integration, the outside one is cosine integration, and the inside one is Bessel integration.

2.2 Transient Electromagnetic Propagation Feature of a Horizontal Layered Earth Medium

Scalar function F satisfies the Helmholz equation in the frequency domain. In Cylindrical coordinates, the general solution of electric and magnetic fields can be expressed as[5]

$$E=M^{\mathrm{D}}\mathrm{e}^{-(m^2-k^2)^{\frac{1}{2}}\cdot z}+N^{\mathrm{U}}\mathrm{e}^{(m^2-k^2)^{\frac{1}{2}}\cdot z}, \tag{14}$$

$$H=A^{\mathrm{D}}\mathrm{e}^{-(m^2-k^2)^{\frac{1}{2}}\cdot z}+B^{\mathrm{U}}\mathrm{e}^{(m^2-k^2)^{\frac{1}{2}}\cdot z}, \tag{15}$$

where M^{D} and A^{D} are down-wave amplitudes, N^{U}, B^{U} are up-wave amplitudes. This general solution contains two parts: positive exponential part and negative exponential part. It can also be regarded as an up-wave part and a down-wave part, as shown in Fig. 2. The Down-wave runs to the earth and decays with depth z following exponential law. Its phase changes with time

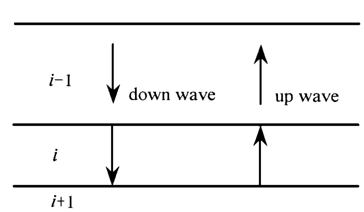

Fig. 2 Up and down waves of the i-th interface in the layered earth

and depth z in a manner of sine function.

When electric and magnetic components are obtained, the wave impedance can be easily derived by a series of calculation[2]

$$Z(0,\omega) = Z_{01}\left[1 + 2\sum_{m=1}^{\infty} q_m \mathrm{e}^{-2\sqrt{-\mathrm{i}\omega t_0}\cdot m}\right] = Z_{01}\left[1 + 2\sum_{m=1}^{m=\infty} q_m p_m\right], \tag{16}$$

where $p_m = \mathrm{e}^{-2\sqrt{-\mathrm{i}\omega t_0}\cdot m}, t_0^2 = \mu_0 \sigma_m h_m^2, t_0$ is the pseudo time parameter, h_m is the depth of the m-th layer, $q_m = (\sqrt{\rho_{m+1}} - \sqrt{\rho_m})/(\sqrt{\rho_{m+1}} + \sqrt{\rho_m})$ is the reflection coefficient of the m-th interface, ρ_m is the m-th layer resistivity, and σ_m conductivity of the m-th layer.

3 NUMERICAL CALCULATION METHOD OF TRANSIENT ELECTROMAGNETIC IMAGING

From the above theoretical deduction, we known that transient electromagnetic method imaging numerical calculation mainly contains two steps. The first step is getting the integral kernel function that is also named normalized wave-impedance with field magnetic data. The second step is getting reflection coefficients from calculated wave impedance. In numerical calculation, linear digital filter technology is used at first step. In this step authors put forward the conception of equivalent wave -impedance. The second step uses the linear programming method. There are transformation from time-domain to frequency domain and from frequency to time domain. In the frequency domain, we get integral kernel function and transform it to the time domain. In the time domain reflection coefficients can be calculated at last.

3.1 Numerical Calculation Method of Getting Wave-Impendence

In this paper, wave impedance is obtained from Eq.(10) with the linear filter method. In order to ensure correctness of result and reduce amount of calculation, we rewrite the integral kernel of Eqs.(11) and (12) as[6]

$$\lambda\left(\frac{Z^{(1)}}{Z^{(1)} + Z_0} - \frac{1}{2}\right),$$

so Eq.(12) becomes

$$H_z(t) = Ia\int_0^{\infty} \lambda\left(\frac{Z^{(1)}}{Z^{(1)} + Z_0} - \frac{1}{2}\right)\mathrm{J}_1(\lambda a)\mathrm{d}\lambda + \frac{1}{2}Ia\int_0^{\infty} \lambda \mathrm{J}_1(\lambda a)\mathrm{d}\lambda, \tag{17}$$

and

$$\frac{1}{2}Ia\int_0^{\infty} \lambda \mathrm{J}_1(\lambda a)\mathrm{d}\lambda = \frac{I}{2a} = H_z^0, \tag{18}$$

where H_z^0 is the original field around the loop center. And we have

$$\frac{H_z(t)}{H_z^0} = 1 + 2a^2\int_0^{\infty} \lambda\left(\frac{Z^{(1)}}{Z^{(1)} + Z_0} - \frac{1}{2}\right)\mathrm{J}_1(\lambda a)\mathrm{d}\lambda, \tag{19}$$

$$\frac{\mathrm{d}B(\omega)}{\mathrm{d}\omega} = -\mathrm{i}\omega\mu_0 H_z. \tag{20}$$

After discretizing Eq.(17), we have

$$\frac{\mathrm{d}B(\omega_j)}{\mathrm{d}\omega} = -\mathrm{i}\omega_j\mu_0\left[1 + 2a^2\sum_{n=-25}^{33} F_1(n\Delta)H_{1n}\right], \tag{21}$$

where ω_j is the j-th frequency, H_{1n} is first order Hankel coefficients from reference [7]:

$$F_1(n\Delta) = \lambda\left(\frac{Z^{(1)}}{Z^{(1)} + z_0} - \frac{1}{2}\right)\bigg|_{\lambda=\mathrm{e}^{n\Delta}}, \tag{22}$$

where $\Delta = \dfrac{\ln 10}{10}$ is the interval of sampling.

Let
$$\frac{Z^{(1)}}{Z^{(1)} + Z_0} - \frac{1}{2} = R(\omega),$$
where $R(\omega)$ is the equivalent wave impedance of uniform half-space when frequency is ω. Eq.(21) can be rewritten as

$$\frac{\mathrm{d}B(\omega_j)}{\mathrm{d}\omega} = \mathrm{i}\omega_j\mu_0\left[1 + 2a^2 R(\omega_j)\sum_{n=-25}^{33} \mathrm{e}^{n\Delta} H_{1n}\right], \tag{23}$$

Eq.(23) is the discrete one that has been processed by the filter method. It also shows that the integral kernel function $R(\omega_j)$ can be solved from Eq.(23), because the transient electromagnetic method still belongs to volume exploration. Wave-impedance of some time delay is a total wave impedance of some depth scope rather than some layer. So the wave-impedance sequence can be replaced by the equivalent wave impedance of respective uniform half space, and transmit matrix can be expressed as a single sequence. Fig. 3 is the flow chart of equivalent wave-impedance calculation.

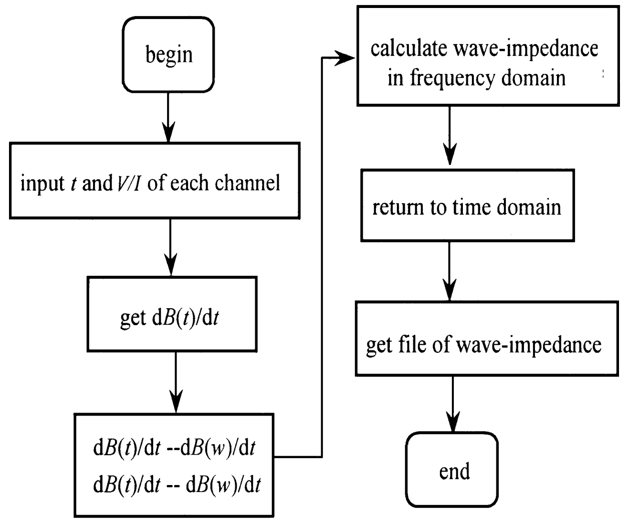

Fig. 3 Flow chart of equivalent wave-impedance calculation

3.2 Numerical Calculation Method of Reflection Coefficient Sequence

Eq.(17) is an underdetermined equation that contains reflection function sequence. It can be solved by the linear programming method. At first, the underdetermined system of the reflection coefficient sequence q_m is constructed. By Fourier transformation, Eq.(23) can be written as[7,8]

$$\begin{cases} R_1 = P_{11}q_1 + P_{12}q_2 + \cdots + P_{1N}q_N, \\ R_2 = P_{21}q_1 + P_{22}q_2 + \cdots + P_{2N}q_N, \\ \qquad \cdots \\ R_M = P_{M1}q_1 + P_{M2}q_2 + \cdots + P_{MN}q_N, \end{cases} \tag{24}$$

where
$$P_{m,i} = -\frac{1}{2\pi\mathrm{i}}\int \frac{\mathrm{e}^{-2\sqrt{-\mathrm{i}\omega_j t_0}\cdot m}}{\omega_j}\mathrm{e}^{-\mathrm{i}\omega_j^t}\mathrm{d}\omega,\ \ R_j = -\frac{1}{2\pi\mathrm{i}}\int R(\omega_j)/\omega\mathrm{e}^{-\mathrm{i}\omega_j^t}\mathrm{d}\omega.$$

The method of linear programming means that under the condition of given equations or inequalities the minimum of some object function is found. It can be expressed as

$$\begin{cases} (R_j + \Delta R_j) \geq \sum\limits_{m=1}^{N} q_m p_{m,j}, \\ (R_j - \Delta R_j) \leq \sum\limits_{m=1}^{N} q_m p_{m,j}, \end{cases} \tag{25}$$

$$f_{\min} = |q_1| + |q_2| + \cdots + |q_N|, \tag{26}$$

where ΔR_j is a mini quantity, which equals to the 5% of R_j during calculation in this paper, and $f_{\min}$ is the sum of absolute values of reflection coefficients.

Considering the requirement of constraint condition and target function of linear programming, there are still some problems during using linear programming:

(1) Linear programming requests that unknown items must be greater that zero, but electromagnetic wave reflection coefficients maybe less than zero or greater than zero, $-1 \le q_m \le 1$.

(2) Linear programming requests that the right item must be positive.

(3) Linear programming can be only used in the real domain, but all of the constraint conditions are complex numbers.

In order to make the geophysical problem satisfy the conditions of linear programming, it must be revised that:

(1) Let $q_1 = u_1 - v_1, q_2 = u_2 - v_2, \cdots, q_m = u_m - v_m$, both $u_1, u_2, \cdots, u_m$, and $v_1, v_2, \cdots, v_m$ are positive constants. It means $u_m \ge 0, v_m \ge 0$, so Eq.(26) can be written as

$$f_{(\min)} = u_1 + v_1 + u_2 + v_2 + \cdots + u_m + v_m, \tag{27}$$

(2) If $R_j \pm \Delta R_j \le 0$, both sides of the inequality are multiplied by -1.

(3) If $R_j \pm \Delta R_j$ is a complex number, both sides of the equation are separated into real part and imaginary part, so inequation (25) becomes

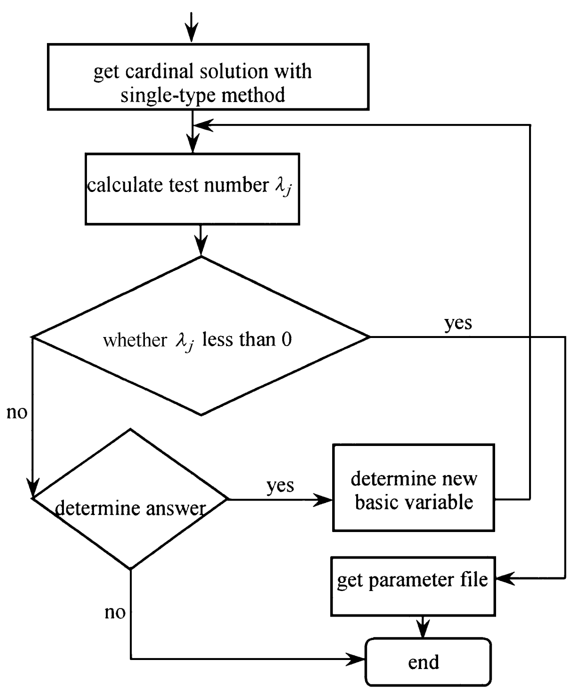

Fig. 4 Flow chart of reflection coefficient calculation

$$(R_j^{\mathrm{R}} + \Delta R_j^{\mathrm{R}}) \ge \left(\sum_{m=1}^{N} (u_m - v_m) p_{m,j}\right)_{\mathrm{R}},$$

$$(R_j^{\mathrm{I}} + \Delta R_j^{\mathrm{I}}) \ge \left(\sum_{m=1}^{N} (u_m - v_m) p_{m,j}\right)_{\mathrm{I}},$$

$$(R_j^{\mathrm{R}} - \Delta R_j^{\mathrm{R}}) \le \left(\sum_{m=1}^{N} (u_m - v_m) p_{m,j}\right)_{\mathrm{R}},$$

$$(R_j^{\mathrm{I}} + \Delta R_j^{\mathrm{I}}) \le \left(\sum_{m=1}^{N} (u_m - v_m) p_{m,j}\right)_{\mathrm{I}},$$

where R is the real part and I is the imaginary part.

The problem searching the reflection function sequence of linear programming becomes the mathematical question of looking for the minimum of target function (27) under the condition of last four inequalities, which are calculated through programming. The flow chart of reflection coefficient calculation is shown in Fig. 4

4 THEORETICAL MODELS AND CALCULATION RESULTS

We compile a computer program according the above idea, and make images to five theoretical modals. The five selected theoretical modals are D-type geoelectric section, Q-type geoelectric section, K-type geoelectric section, H-type geoelectric section and A-type geoelectric section. In Fig. 5a, the fist layer is separated into five mini-layers ($t_0 = t(h_1/5)$, therefore the original reflection appears at the time of $10t_0$. In Fig. 5b, the first layer is regarded as one mini-layer ($t_0 = t(h_1)$, so the original reflection of each interface appears at the time of $2t_0$ and $12t_0$, respectively. In Fig. 5c, the first layer is regarded as a mini-layer $t_0 = t(h_1)$, so the original reflection of two interfaces appears at the time of $2t_0$ and $8t_0$, the respectively. In Fig. 5d, the first layer is separated into two mini-layers ($t_0 = t(h_1/2)$), then the original reflection of each interface appears at the time of $4t_0$ and $14t_0$, respectively. In Fig.5e, the first layer is regarded as a mini-layer ($t_0 = t(h_1)$), then original reflection of two interfaces appears at the time of $2t_0$ and $6t_0$, Every modal is processed to transform from time to depth. The transformed results show that there is consistence between calculation results and given parameters. During the calculation, we introduce a kind of skill of calculation, that is calculating each frequency at first, then studying revised reflection coefficients $\{q_m\}$ on the base of equivalent wave-impedance $R(\omega)$. To some extent the method of calculation really means to get average of reflection coefficients, and the calculated parameter is suited to qualitative analysis more, though there is difference from the real reflection coefficients of underground electric interface. After all the method can depress multi-wave effectively, give prominence to original reflection and make section clear.

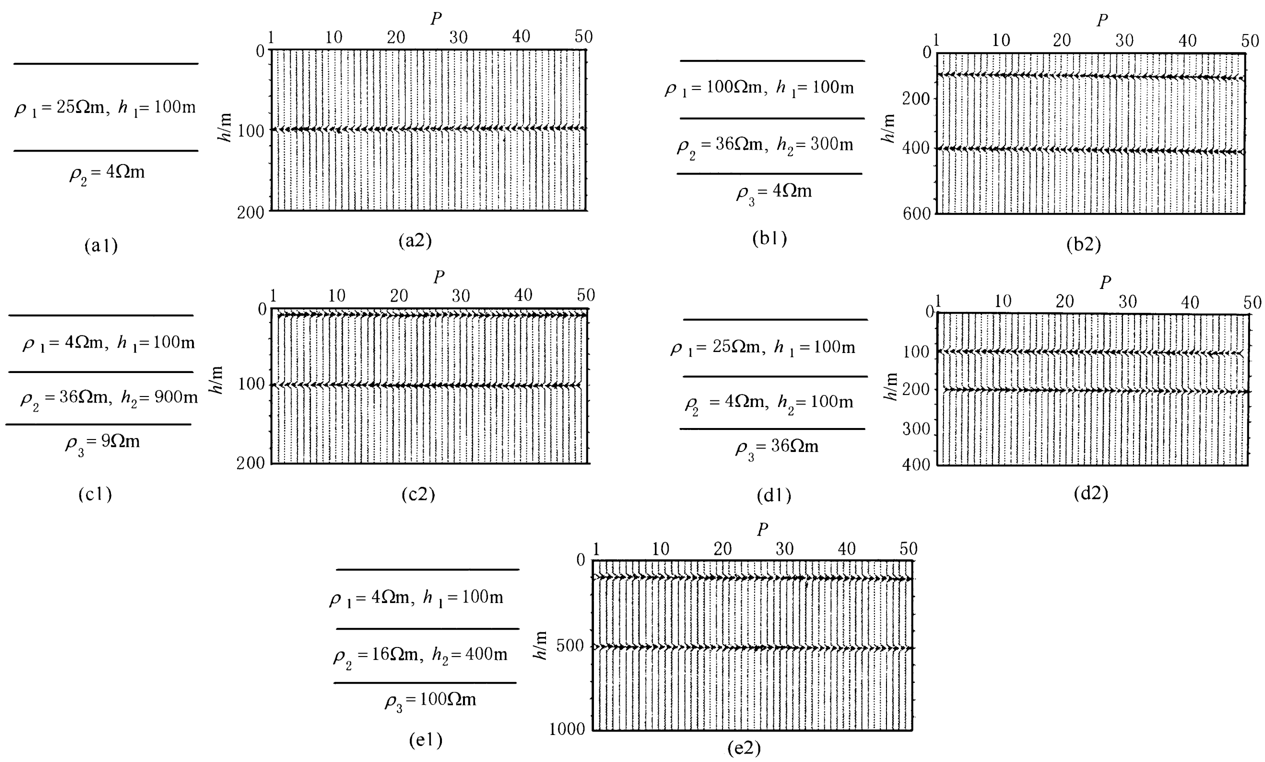

Fig. 5 Imaging sections of variation type

(a), (b), (c), (d) and (e) are two-layered D-type, three-layered Q-type, three-layered K-type, three-layered H-type and three-layered A-type, respectively; a1, b1, c1, d1 and e1 are earth electric sections; a2, b2, c2, d2 and e2 are imaging sections.

5 CONCLUSIONS

We analyze theoretically the loop-source transient electromagnetic imaging method, discuss the response of horizontal layered medium-electromagnetic medium, and derive a double integral equation that contains unknown wave-impedance. The electromagnetic field solution of layered earth response equation is separated into up- and down -traveling waves, and the linear functions that contain reflection coefficient sequence are obtained. A numerical calculation method of reached functions is put forwarded that replaces integral Kernel function by equivalent wave-impedance of homogenous half-space. Then it separates electromagnetic response integral equation, gets equivalent wave-impedence from field data of magnetic induction, constructs the equations of reflection coefficients, and solves reflection coefficient with linear programming. Finally we draw imaging sections with the calculated parameter. The method of getting revised reflection coefficient system can directly reduce the amount of calculation, and depress multi-wave effectively. The result demonstrates that the idea given in this paper is feasible in numerical simulation. The processed sections directly show the features underground electric interfaces.

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REFERENCES

[1] Li X. The Theory and Application of Transient Electromagnetic Sounding. Xi'an: Shanxi Science Technology Press, 2002, 1∼4, 102∼105

[2] Wang J Y. Magnetotelluric Pseudo-Seismic Interpretation. BeiJing: Petroleum Industry Press, 1990, 30∼41

[3] Chen B C, Zhou F T, Li J M. The wavefield transformations study of transient electromagnetic field. *Geophysical & Geochemical Exploration*, 1999, **23**(3): 195∼201

[4] Fang W Z, Li Y G, Li X. Theory of Transient Electromagnetic Sounding. Xi'an: North-West Industry University Press, 1993. 9∼15

[5] Alexander A Kaulfman, GEOGER V Keller, Wang J M translated. Frequency and Transient Soundings. Beijing: Geology Publishing House, 1987. 18∼19

[6] Piao H R. The Theory of Electromagnetic Sounding. Beijing: Geology Publishing House, 1990. 151∼161

[7] Xue G Q. The study of transient electromagnetic imaging [Master thesis]. Xi'an: School of Geology and Survey, Chang'an University, 2002

[8] Xue G Q, Song J P, LI X. Determing the reflection coefficients of TEM field by the method of linear programming. *Journal of Xi'an Engineering University*, 2002, **24**(3): 72∼75